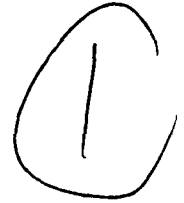
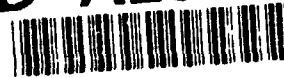


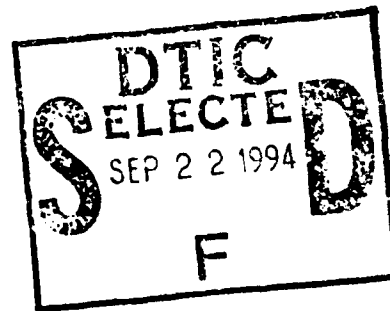
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# Variational Principle Modeling of Class IV Flextensional Transducers

A Paper Presented at the 126th Meeting of the  
Acoustical Society of America, October 1993,  
Denver, Colorado

Harold C. Robinson  
Environmental and Tactical Support Systems Department



**Naval Undersea Warfare Center Division**  
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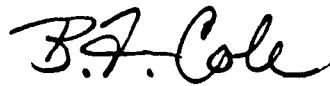


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## **PREFACE**

The research presented in this document was sponsored by the Office of Naval Research and was presented at the 126th meeting of the Acoustical Society of America.

**Reviewed and Approved: 21 June 1994**

A handwritten signature in cursive script, reading "B. F. Cole". The signature is written in black ink and is centered below the approval date.

**B. F. Cole**  
**Head, Environmental and Tactical Support**  
**Systems Department**

**REPORT DOCUMENTATION PAGE**Form Approved  
OMB No. 0704-0188

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1. AGENCY USE ONLY (Leave Blank)		2. REPORT DATE  21 June 1994	3. REPORT TYPE AND DATES COVERED  Final	
4. TITLE AND SUBTITLE  Variational Principle Modeling of Class IV Flextensional Transducers			5. FUNDING NUMBERS	
6. AUTHOR(S)  Harold C. Robinson				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  Naval Undersea Warfare Center Detachment 39 Smith Street New London, Connecticut 06320-5594			8. PERFORMING ORGANIZATION REPORT NUMBER  TD 10, 671	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  Office of Naval Research 800 North Quincy Street Arlington, VA 22217-5000			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES  A Paper Presented at the 126th Meeting of the Acoustical Society of America, October 1993, Denver, Colorado				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Approved for public release; distribution unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  This document contains the material from a presentation entitled "Variation Principle Modeling of Class IV Flextensional Transducers," presented at the 126th Meeting of the Acoustic Society of America.				
14. SUBJECT TERMS			15. NUMBER OF PAGES 13	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT  Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE  Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT  Unclassified	20. LIMITATION OF ABSTRACT  SAR	

# VARIATIONAL PRINCIPLE MODELING OF CLASS IV FLEXTENSIONAL TRANSDUCERS

## INTRODUCTION

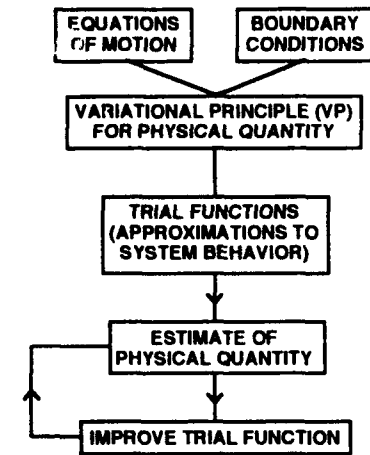
One approximation method that can accurately estimate a quantity of interest using relatively crude representations for the physical behavior of the system is the variational principle technique. This modeling method is applied to flextensional transducer analysis by coupling a variational principle developed for the resonance frequency of the piezoelectric driving element to one for the resonance frequency of the shell, carefully ensuring that the boundary conditions at the driver-shell interface are satisfied. The *in-vacuo* mode shapes and resonance frequencies for a Class IV transducer calculated in this manner are compared with finite element models and experimental data for the first two even-even modes of the transducer. There is good agreement between the methods in the calculation of the resonance frequencies of these modes, even though the mode shapes calculated variationally do not agree exactly with the finite element predictions.

Fluid-loading effects on the transducer are introduced by coupling the *in-vacuo* variational transducer model to a variational principle for the surface pressure on a radiating body. This surface variational principle is derived from the Helmholtz integral equation. The surface pressures determined using this variational formulation and the finite element method for a single Class IV transducer are compared for the first two even-even resonant modes. It is shown that the agreement for the surface pressure is better for the first mode, but in both cases the agreement is still reasonable. Possible reasons for these discrepancies are discussed, as well as options for calculating the far-field pressures from the variationally determined surface pressures.

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## VARIATIONAL PRINCIPLES

**OBJECTIVE: TO OBTAIN ACCURATE ESTIMATES OF PHYSICAL QUANTITIES WITHOUT THE CONSTRAINT OF SATISFYING THE EQUATIONS OF MOTION OR BOUNDARY CONDITIONS EXACTLY**



**EXAMPLE: RESONANCE FREQUENCY OF CIRCULAR MEMBRANE**

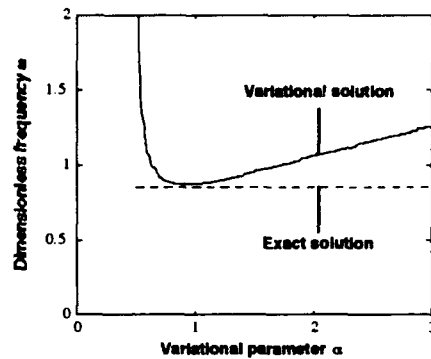


FIGURE 1

A variational principle (VP) provides accurate estimates of some physical quantity without having to satisfy either the equations of motion or the boundary conditions exactly. The equations of motion and the boundary conditions are combined using the generalized method of E. Gerjuoy, A.R.P. Rau and L. Spruch ["A Unified Formulation of the Construction of Variational Principles," *Rev. Mod. Phys.* 55, pp. 725-774, (1983)] into a variational expression. Approximations to the behavior of the system, or trial functions, are incorporated into the VP to yield the estimate. The accuracy of this estimate can be improved by refining the trial functions. This is accomplished either by allowing the trial functions to satisfy some of the boundary conditions or by adding more terms to the trial function (Rayleigh-Ritz method).

An example of a typical variational calculation is shown on the right. The lowest dimensionless resonance frequency  $\omega$  for the axisymmetric motion of a circular membrane is well known, as is the exact solution for the displacement (a zeroth order Bessel function) [A.L. Fetter and J.D. Walecka, *Theoretical Mechanics of Particles and Continuous Media*, McGraw-Hill, New York, pp. 219-244 (1980)]. Nonetheless, it is illustrative to calculate the resonance frequency variationally, using a displacement of the form  $\cos(\alpha r)$ , where  $r$  is the radius of the membrane, and varying the parameter  $\alpha$ . The best variational solution occurs when the minimum frequency is reached. The frequency estimate is very good, even though the trial function is not an exact representation of the true displacement. This fundamental property of VPs makes them attractive for calculations involving complex physical systems.

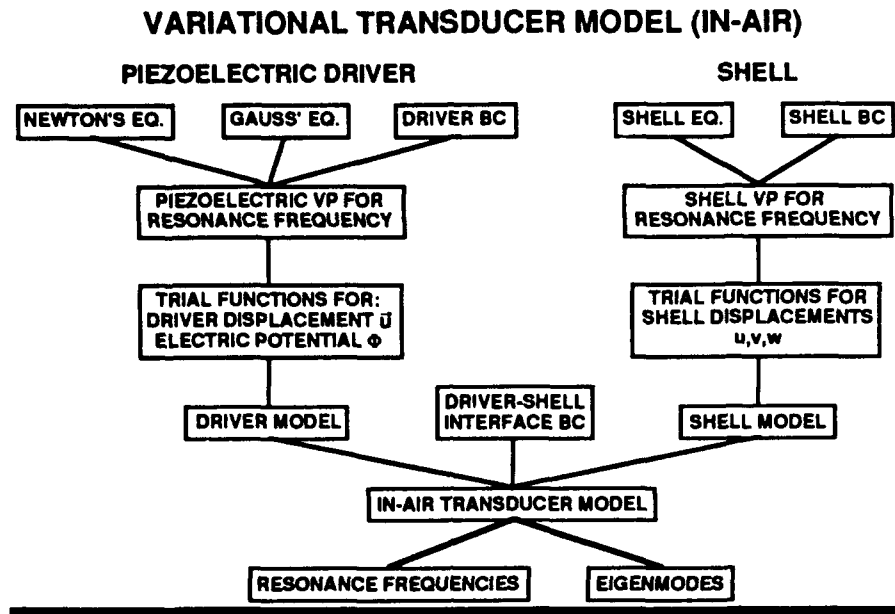


FIGURE 2

The development of a variational model for flextensional transducers is illustrated above. The primary constituents of such a transducer are a driver, usually consisting of piezoelectric ceramic, attached to an elastic shell. The extensional motion of the driver causes the shell to move in a flexural mode, hence the term flextensional. To obtain a variational model for the transducer as a whole, variational expressions for the resonance frequency of the individual parts are first derived. Newton's and Maxwell's equations for the piezoelectric material, expressed in the notation of B.A. Auld [*Acoustic Fields and Waves in Solids*, Wiley-Interscience, New York (1973)], are combined with appropriate electrical and mechanical boundary conditions to yield a VP for the resonance frequency of the driver. The appropriate independent variables for this VP are the driver displacement  $\bar{u}$  and the electric potential  $\Phi$ . Similarly, shell equations, such as the generalized equations in A.W. Leissa ["Vibration of Shells," NASA Sp-288, Washington, D.C., 1973], and boundary conditions, e.g. free, clamped, etc., yield a VP in terms of the normal shell displacement  $w$  and two tangential displacements  $u$  and  $v$ . The models for the piezoelectric driver and elliptical shell for a Class IV transducer have been verified independently against experimental data and previous models [G.A. Brigham, "In-plane Free Vibrations of Tapered Oval Rings," *J. Acoust. Soc. Am.* **54**, pp. 451-460 (1973)]. Once the individual VPs have been verified, they may be combined, using interface boundary conditions, into a variational model for the transducer which yields highly accurate estimates for the eigenfrequencies but less accurate mode shapes.

### CLASS IV GEOMETRY

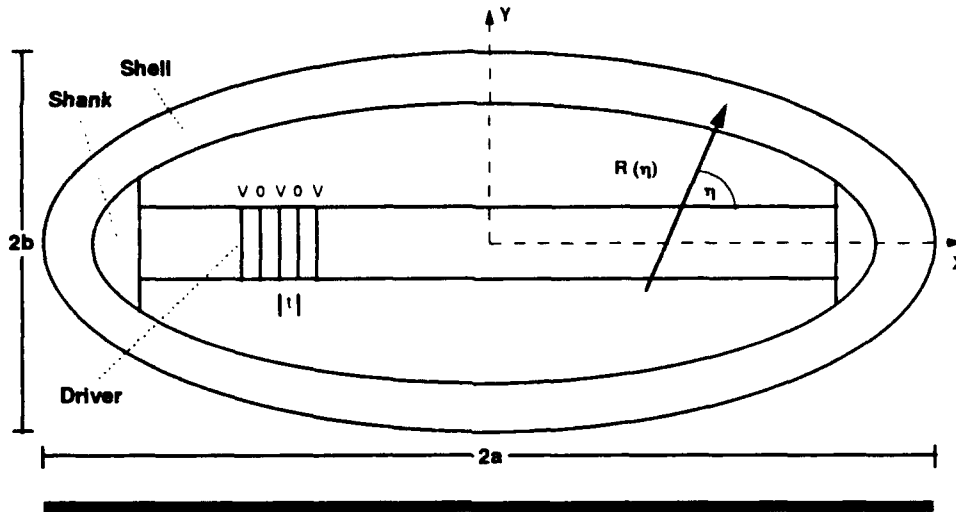


FIGURE 3

The simplified model geometry for the Class IV flextensional transducer is shown here. The transducer consists primarily of a piezoelectric stack driver and an elliptical or oval shell. The driving stack consists of rectangular slabs of piezoelectric ceramic with thickness  $t$ . The slabs are electroded on both ends, with the voltage alternating so that each pair of slabs is expanding along the  $x$  axis. This requires that the piezoelectric material constants must be rotated properly to ensure the correct poling direction is maintained. The motion and electric potential of the driver are modeled in Cartesian coordinates  $(x, y, z)$ . As a first-order approximation, we can ignore the dependence of the stack motion on the coordinates  $y$  and  $z$ , leaving only pure extensional motion along the  $x$  axis. Insulators at the ends of the stack are included as purely elastic constituents. The driver is connected to the shell by a metallic shank which, for modeling purposes, is considered to be an additional mass on the driver.

The shell is a metallic elliptic cylinder, with a semi-major axis length  $a$ , a semi-minor axis length  $b$  and uniform thickness  $h$ . The shell motion is modeled in elliptical cylinder coordinates  $(\xi, \eta, z)$  relative to the shell's midsurface where

$$x = c \cosh \xi \cos \eta \quad y = c \sinh \xi \sin \eta \quad z = z \quad c = \sqrt{a^2 - b^2}$$

The shell motions depend only on the coordinate  $\eta$ , which gives the angle between the local radius of curvature  $R(\eta)$  of the shell midsurface and the  $x$  axis. Thus, at least formally, the shell motions are independent of the axial ( $z$ ) and thickness ( $\xi$ ) coordinates, and the axial motion is uniquely zero.

## TRIAL FUNCTIONS FOR DRIVER AND ELLIPTICAL SHELL

### ● TRIAL FUNCTIONS FOR PIEZOELECTRIC STACK DRIVER

$$u_d = \sum_{n=1}^{\infty} U_{dn} \left( \frac{x}{t} \right)^{2n+1} \quad \text{driver displacement}$$

$$\Phi = \sum_{n=1}^{\infty} \Phi_n \cos^2 \left[ \frac{(2n+1)\pi x}{2t} \right] \quad \text{electric potential}$$

### ● TRIAL FUNCTIONS FOR ELLIPTICAL SHELL DISPLACEMENT

$$V = v + \zeta \psi_{\eta} \quad W = w + \zeta \psi_z$$

IN THE FORM

$$v = \sum_{n=1}^{\infty} [V_n^o \cos(n\eta) + V_n^e \sin(n\eta)]$$

ETC.

FIGURE 4

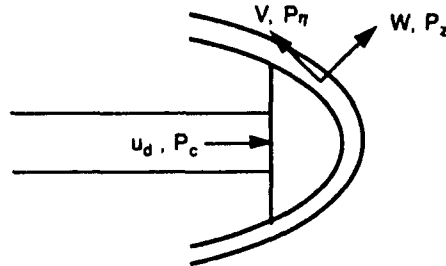
The trial functions for the driver and shell are detailed here. The driver displacement along the x axis is described by an odd power series in the Cartesian coordinate x, which ensures that the two ends of the driver move equally and in opposite directions. The potential is described as a cosine squared series, which gives the trial functions a value of 0 where the actual potential is zero and a maximum value where the real potential is  $V_0$ .

The trial functions for the shell are typical of most geometric shell theories. The form presented here is consistent with thick shell theory for circular cylindrical shells [I. Mirsky, "Vibrations of Orthotropic, Thick Cylindrical Shells," J. Acoust. Soc. Am. 36, pp. 41-51 (1964)]. The total normal displacement W is the sum of a normal displacement w, dependent only on the angle  $\eta$ , and a transverse normal shear  $\psi_z$ , also dependent only on the angle  $\eta$ , multiplied by  $\zeta$ , the thickness coordinate normal to the midsurface of the shell. Similarly, the total azimuthal displacement V is the sum of a tangential displacement v and a change in curvature  $\psi_{\eta}$  multiplied by  $\zeta$ . From the theory of elliptical rings [K. Sato, "Free Flexural Vibrations of an Elliptical Ring in its Plane," J. Acoust. Soc. Am. 57, pp. 113-115 (1975)], it is known that the motion of the shell splits into four distinct categories that can be characterized in terms of their symmetry about the major and minor axes as even-even, even-odd, etc. To represent this, we parametrize the motion of the shell using trial functions which are odd and even (denoted by the superscripts o and e) in the tangential coordinate  $\eta$ .



## IN-AIR COUPLING OF DRIVER AND SHELL

- PIEZOELECTRIC VARIATIONAL PRINCIPLE AND SHELL VARIATIONAL PRINCIPLE MUST BE SOLVED SIMULTANEOUSLY UNTIL CONVERGED RESULT FOR THE RESONANT FREQUENCY IS OBTAINED
- BOUNDARY CONDITIONS AT SHELL-DRIVER INTERFACE



$$V(\eta, z = -h/2) = -\frac{u_d(x=a)}{\sqrt{1-e^2 \cos^2 \eta}} \sin \eta$$

$$W(\eta, z = -h/2) = \frac{u_d(x=a)}{\sqrt{1-e^2 \cos^2 \eta}} \frac{b}{a} \cos \eta$$

FIGURE 5

As discussed previously, the driver and shell boundary conditions must be coupled to give a complete flextensional transducer model. It is therefore important to ensure that the boundary conditions at the shell-driver interface are satisfied as well as possible. These conditions are shown in the bottom illustration [G.A. Brigham, "Analysis of the Class-IV Flextensional Transducer by Use of Wave Mechanics," J. Acoust. Soc. Am. 56, pp. 31-39 (1974)]. The shank is assumed to move in unison with the driver, and flexure of the shank is neglected; thus, the driver displacement  $u_d$  is translated through the shank to the interior surface of the shell. We can then match the shell displacements with the driver displacements at the interface surface as detailed in the lower equations, where  $e$  is the eccentricity of the midsurface of the shell. The normal driver stress  $P_c$  is similarly matched to the normal and shear shell stresses as well as possible. Because it is, in general, not possible to match both stresses and displacements simultaneously, any unmatched stress contributions are compensated for via additional work terms in the variational principle for the transducer's resonance frequency.

### MODE SHAPES OF THE TWO LOWEST EVEN-EVEN FLEXURAL RESONANCES

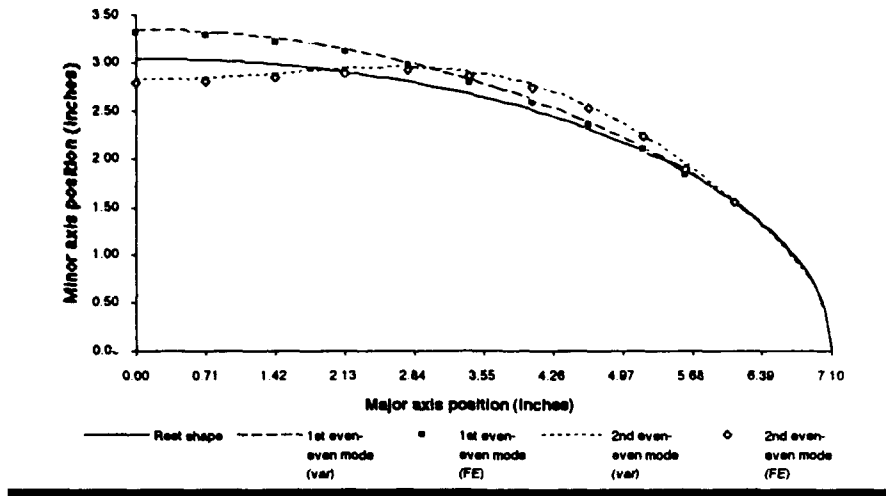


FIGURE 6

The first two variationally determined even-even flexural mode shapes are shown here as a function of position along the main (x) axis. The variational calculations are compared to finite element results provided by Dr. Rick Morrow of General Electric Corporate Research and Development, Schenectady, New York. The modal displacements are exaggerated by a factor of 400 in order to illustrate the behavior. The first mode agreement is very good across the surface of the shell and clearly represents a flexural mode where the motion is dominated by the normal component of the shell displacement. The difference between the finite element and variational results for the first mode shape is less than 5%. The second mode is not represented as well, which is to be expected. The second even-even mode is clearly a higher order flexural mode of the shell. The variational principle does not pick up the additional bending in the second mode which occurs between five and six inches, although this does not appear to be a major discrepancy. As anticipated, the largest displacement is along the minor axis of the shell for both modes, with very little motion along the major (driver) axis. Although in practice the transducer consists of three shells, each with a length of 3.2 inches, stacked on top of each other, we are neglecting the dependence of the resonant modes on the length. This is justifiable since the lowest modes do not appear to have much axial motion associated with them.

**RESONANCE FREQUENCIES FOR CLASS IV (MOD 30) IN AIR**

RESONANCE FREQUENCIES (HZ)			
	1ST EVEN-EVEN	2ND EVEN-EVEN	BREATHING
VARIATIONAL	1732	5408	5927
2D FINITE ELEMENT (NUSC)	1786	5441	6233
3D FINITE ELEMENT (GE)	1657	5330	5898
EXPERIMENT	1625	5024	5783

FIGURE 7

While the variational principle yields results for the mode shapes which are fairly good, the results for the resonance frequencies should be (and are) even better. This illustrates the fundamental property of variational methods, that the variational principle will yield results which are more accurate than the trial functions used to obtain them. Here the variational resonance frequencies for a Sanders Model 30 transducer in air are compared to 2 and 3 dimensional finite element calculations (performed at NUSC in 1990 by Susan LaShomb using the program FEDESC) and experimental values. The variational calculations appear to be better than the 2D finite element model but not as good as the 3D model. Discrepancies between the calculations could be due to minor differences between the NUWC finite element, GE finite element, and variational material parametrizations and geometries. The NUWC model uses an older Mod 30 configuration than either the GE or variational models; the GE model also includes losses not present in the others. It should also be noted that the experimental values quoted are merely typical for this model of Class IV transducer; there is some variation in these values depending on which specific transducer was measured or what method of measurement was used [M.B. Moffett and R.S. Janus, "Flexensional Acoustic Projector Nearfield Measurements," NUSC Technical Memorandum No. 861171 (15 Sept. 1986); C.J. Dubord and D.L. Walters, "Surface Ship Hull/Transducer Interaction Evaluation," NUSC Technical Memorandum No. 871068 (23 March 1987)]. Nevertheless, the variational results do fall with the range of available experimental measurements for this transducer.

## VARIATIONAL FLUID LOADING FORMULATION

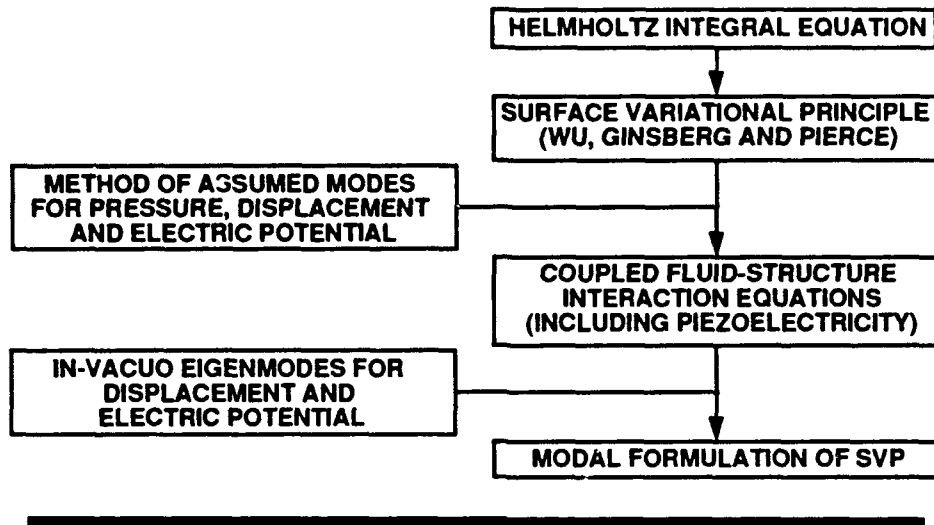


FIGURE 8

A variational formulation for fluid-loading effects begins with the Helmholtz integral equation [M.C. Junger and D. Feit, Sound, Structures and Their Interaction, 2nd ed., MIT Press, Cambridge, MA (1986)], which relates the sound pressure in a fluid to the velocity and pressure distribution on the surface of a radiating body. X.F. Wu ["Variational Principles for Acoustic Radiation and Diffraction from Underwater Structures," Georgia Institute of Technology Rept. No. GTADL-TR-87-102, 24 November 1987] and J.H. Ginsberg, P.T. Chen and A.D. Pierce ["Analysis Using Variational Principles of the Surface Pressure and Displacement Along an Axisymmetrically Excited Disk in a Baffle," *J. Acoust. Soc. Am.* **88**, pp. 548-559 (1990)] developed a stationary expression for the pressure on the surface of a radiating body with a prescribed surface velocity by allowing the field point in the Helmholtz integral equation to approach the surface and carefully taking the limits of the integrals involved. Structural effects are incorporated using the so-called method of assumed modes, in this case a basis function expansion for the pressure, displacement and electric potential of the transducer. This yields a set of coupled fluid-structure equations reminiscent of finite element formulations [R.R. Smith, J.T. Hunt and D. Barach, "Finite Element Analysis of Acoustically Radiating Structures with Applications to Sonar Transducers," *J. Acoust. Soc. Am.* **54**, pp. 1277-1285 (1973)], except that the structure has not been discretized. The complexity of the problem may be further reduced by using previously determined *in-vacuo* eigenmodes for the structural displacement, leading to a modal formulation of the surface variational principle (SVP).

## FLUID LOADING IN CLASS IV TRANSDUCER MODEL

- EXPRESS DISPLACEMENTS AND ELECTRIC POTENTIAL IN TERMS OF IN-VACUO EIGENMODES OF CLASS IV TRANSDUCER

$$\bar{U}(\bar{x}) = \sum_{m=1}^L \chi_m \bar{U}_m(\bar{x}) \quad \Phi(\bar{x}) = \sum_{m=1}^L \chi_m \phi_m(\bar{x})$$

- TRIAL FUNCTIONS FOR SURFACE PRESSURE

$$p = \sum_{n=1}^N \left[ P_n^o \cos(n\eta) + P_n^o \sin(n\eta) \right]$$

- MODAL FORMULATION OF SVP

$$\begin{bmatrix} [\omega_n^2 - \omega^2] & [A] \\ -[\Gamma] & [A] \end{bmatrix} \begin{bmatrix} \{\chi\} \\ \{P\} \end{bmatrix} = \begin{bmatrix} \{F_m\} \\ \{F_p\} \end{bmatrix}$$

FIGURE 9

The variationally determined eigenmodes of the transducer are denoted by  $\bar{U}_m$  (for the combination of the shell and driver displacements) and by  $\phi_m$  (for the electrostatic potential). Any arbitrary displacement and potential can now be expressed as a linear combination of these modes. Because the *in-vacuo* eigenmodes of the transducer contain shell motions which are of even and odd symmetry about the major axes, we expand the surface pressure in even and odd functions of the angle  $\eta$  as before. Combining these expansions with the coupled fluid-structure interaction equations yields the modal formulation of the surface variational principle, where  $\{\chi\}$  and  $\{P\}$  denote the set of expansion coefficients for the *in-vacuo* eigenmodes and the surface pressure, respectively.  $\omega_n$  is the *in-vacuo* resonance frequency of the transducer. The matrices  $[A]$  and  $[\Gamma]$  provide the coupling between the transducer motion and the fluid, while the matrix  $[A]$  gives the pressure mode coupling contributions.  $\{F_m\}$  and  $\{F_p\}$  represent external forcing functions. The solution of this matrix equation for a given frequency  $\omega$  yields the variationally determined surface pressure of the radiating body.

# SURFACE PRESSURE ON CLASS IV TRANSDUCER (1ST EVEN-EVEN MODE)

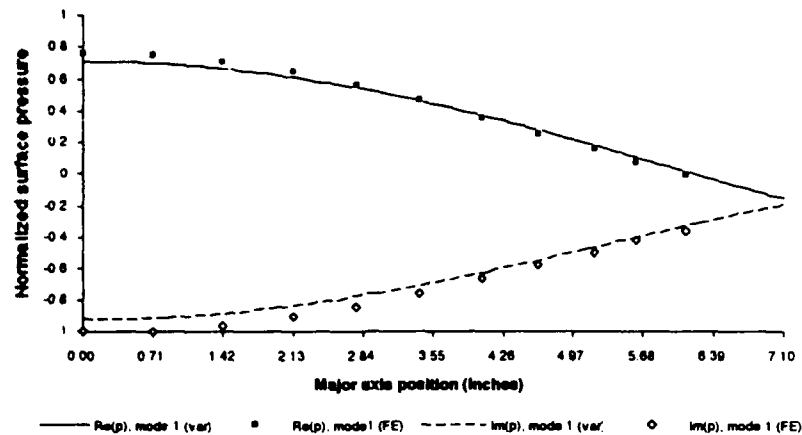


FIGURE 10

A comparison of the variationally computed surface pressures to finite element calculations provided by GE/CR&D is shown for the first even-even mode (ca. 1 kHz). The pressure has been normalized to the maximum magnitude of the imaginary pressure component calculated by the finite element method along the minor axis. As can be seen, there is very good agreement between the two calculations, except for minor differences in the magnitude of the imaginary part (approximately 5%). The shape of the surface pressure is consistent with the pattern of the surface velocity for the first even-even mode, which is normal to the surface for all angles. Also, the finite element results for the real part indicate a slight change in curvature not seen variationally. Another indication of the validity of the results is that the surface pressure goes to zero as we approach the major axis. This is because there is very little transducer motion at the ends of the shell. This discrepancy might be eliminated with the inclusion of additional trial functions.

**SURFACE PRESSURE ON CLASS IV TRANSDUCER  
(2ND EVEN-EVEN MODE)**

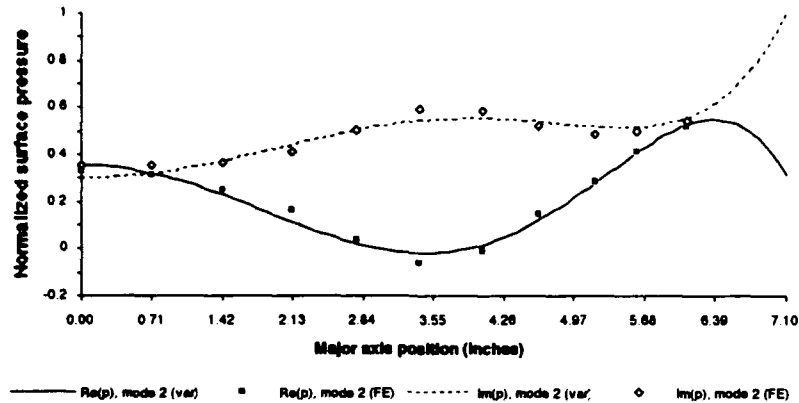


FIGURE 11

The agreement between finite element results and the variational principle remains good for the second even-even mode, although there is some degradation relative to the first mode which is typical of such variational methods. Again, the magnitudes of the variational surface pressure components are within 5% of the finite element results. The curvature for the imaginary part of the surface pressure near the semi-minor axis is much more pronounced in the finite element calculations than in the variational ones. In addition, the curvature for the imaginary part near the semi-minor axis of the shell (major axis position < 0.7 inch) appears to be in the opposite sense. Overall, the agreement is better for the real part of the surface pressure. Finally, it is difficult to assess whether the pressure behavior near the major axis is accurately predicted by the variational principle or not. It would therefore be beneficial to have additional points closer to the major axis in order to determine the veracity of the variational calculations.

## VARIATIONAL PRINCIPLE MODELING OF CLASS IV FLEXTENSIONAL TRANSDUCERS—CONCLUSIONS

- VARIATIONAL PRINCIPLES FOR PIEZOELECTRIC DRIVER AND SHELL ELEMENTS OF CLASS IV FLEXTENSIONAL TRANSDUCERS HAVE BEEN DEVELOPED AND VALIDATED
- COUPLING PIEZOELECTRIC AND SHELL ELEMENTS TO MODEL CLASS IV TRANSDUCER IN AIR YIELDS GOOD RESULTS FOR RESONANCE FREQUENCIES AND REASONABLE RESULTS FOR THE MODE SHAPES
- INCLUDING FLUID LOADING USING SURFACE VARIATIONAL PRINCIPLE FORMULATION GIVES SURFACE PRESSURE VALUES IN GOOD AGREEMENT WITH FINITE ELEMENT PREDICTIONS
- FAR FIELD PRESSURES CAN BE CALCULATED BY USING VARIATIONALLY DETERMINED SURFACE PRESSURES IN BOUNDARY INTEGRAL FORMULATION OR BY MATCHING WITH THE FAR FIELD EXPRESSIONS AT THE SURFACE

FIGURE 12

In conclusion, we can see that a model of the class IV flextensional transducer based on variational principles yields good results for the resonance frequencies of the in-air transducer and the surface pressures on the fluid-loaded transducer. In both cases, the mode shapes calculated are reasonable, although the agreement for the second even-even mode is not as good as the first even-even mode. Possible sources of discrepancy between the variational and finite element treatments are slight differences in the material parameters and in geometry. Also, modeling the shell as an elliptic cylinder is an approximation; however, the geometric difference between an elliptic shell of constant thickness and the actual oval shell of the Class IV transducer is everywhere less than 3%, and the numerical integrations required for the variational calculation are greatly simplified in elliptic cylinder coordinates.

The surface pressures may be utilized to determine the beam pattern of the transducer in two ways. The first possibility is to use these pressures in a boundary integral formulation to obtain the pressure. The second is to match the variational surface pressures with far-field expansions evaluated at the transducer surface. This latter approach might be complicated, since the far-field expansion in elliptical cylindrical coordinates requires the use of Mathieu functions [G.A. Brigham, J.J. Libuha and R.P. Radlinski, "Analysis of Scattering from Large Planar Gratings of Compliant Cylindrical Shells," J. Acoust. Soc. Am. 61, pp. 48-59 (1977)] and must account for the finite transducer length. The transformation of this expansion to spherical coordinates could prove intractable, requiring the use of boundary integral methods anyway.



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